

Quasinormal Modes, Stability Analysis and Absorption Cross Section for 4-dimensional Topological Lifshitz Black Hole

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We study scalar perturbations in the background of a Topological Lifshitz black hole in four dimensions. We compute analytically the quasinormal modes and from these modes we show that Topological Lifshitz black hole is stable. On the other hand, we compute the reflection and transmission coefficients and the absorption cross section and we show that there is a range of modes with high angular momentum which contributes to the absorption cross section in the low frequency limit. Furthermore, in this limit, we show that the absorption cross section decreases if the scalar field mass increases, for a real scalar field mass.

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I. INTRODUCTION

The Lifshitz black holes are interesting because they are solutions of theories that exhibit the anisotropic scale invariance $t \rightarrow \lambda^z t$, $x \rightarrow \lambda x$, where z is the relative scale dimension of time and space¹ and by its representation as the gravitational dual of strange metals [1]. In the present work we consider the four dimensional Topological Lifshitz black holes [2], with dynamical exponent $z = 2$, which is described by the metric

$$ds^2 = -\frac{r^2}{l^2} \left(\frac{r^2}{l^2} - \frac{1}{2} \right) dt^2 + \frac{dr^2}{\left(\frac{r^2}{l^2} - \frac{1}{2} \right)} + r^2 d\Omega^2, \quad (1)$$

where $d\Omega^2 = d\theta^2 + \sinh^2\theta d\phi^2$, and we present exact solutions for the quasinormal modes (QNMs), the reflection and transmission coefficients and the absorption cross section. The QNMs, as the absorption cross section have a long history, see [3], [4], [5], [6], [7], [8], [9] and [10], [11], [12], [13], [14], [15], [16], respectively. The QNMs have been mainly related to the quantized black hole area and mass [17], [18] and [19]. Furthermore, the QNMs have been also related to thermal conformal field theories [20] in the context of AdS/CFT correspondence [21]. In [22] a quantitative test of such a correspondence was made for the BTZ black hole. Another consequence of AdS/CFT correspondence has been the application beyond high energy physics to another areas of physics, for example quantum chromodynamics and condensed matter. In this sense, Lifshitz spacetimes have received great attention from the condensed matter point of view i.e., the searching of gravity duals of Lifshitz fixed points. In [23] has been conjectured gravity duals of non relativistic Lifshitz-like fixed points. This correspondence have been also studied in [1]. In this paper we study the quasinormal modes of scalar perturbations, therefore through this correspondence the quasinormal modes of Lifshitz black holes have an interpretation as the poles of the holographic Green's function related to the bulk scalar field.

On the other hand, the absorption cross section allows us to know the absorption probability and to study the scattering problem. Recently, the QNMs have been reported, as well as the stability analysis and the area spectrum of three dimensional Lifshitz black holes [24] and the absorption cross section [25] and [26], the QNMs and stability analysis of four dimensional Lifshitz black hole with plane transverse section [27] and [28] and the QNMs and thermodynamic quantities for three and four dimensional Lifshitz black holes [29].

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¹ If $z = 1$, the spacetime is the usual AdS metric in Poincare coordinates

In this work, we analyze numerically the behavior of the reflection and the transmission coefficients and the absorption cross section of four dimensional Topological Lifshitz black holes and we find that there is a range of modes with high angular momentum which contributes to the absorption cross section and that it decreases if the scalar field mass increases, for real scalar field masses in the low frequency limit.

This paper is organized as follows. In Sec. II, we consider Dirichlet and Neumann boundary conditions and we calculate the exact QNMs of the scalar perturbations. Then, from these quasinormal modes, we study the stability of this black hole. In Sec. III we calculate the reflection and the transmission coefficients and the absorption cross section. Finally, our conclusions are in Sec. IV.

II. QUASINORMAL MODES

The QNMs of scalar perturbations in the background of a Topological Lifshitz black hole in four dimensions are given by Klein-Gordon equation of the scalar field solution with boundary conditions. This means that there is only ingoing waves on the horizon and the scalar field, Dirichlet boundary condition, or the flux, Neumann boundary condition, vanishes at infinity. The Klein-Gordon equation is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\psi = m^2\psi, \quad (2)$$

where m is the mass of the scalar field ψ , which is minimally coupled to curvature. In order to find an exact solution to the scalar field we will perform the following change of variables $y = \frac{2r^2}{l^2}$, which will allow us to write the metric (1), as

$$ds^2 = -\frac{y}{4}(y-1)dt^2 + \frac{l^2 dy^2}{4y(y-1)} + \frac{l^2 y}{2}d\Omega^2. \quad (3)$$

Now, if we consider the ansatz $\psi = R(y)Y(\Sigma)e^{-i\omega t}$, where Y is a normalizable harmonic function on Σ_{d-2} which satisfies $\nabla^2 Y = -QY$, being

$$Q = \left(\frac{d-3}{2}\right)^2 + \xi^2, \quad (4)$$

the eigenvalues for the hyperbolic manifold. The Klein-Gordon equation (2), can be written as

$$y(1-y)\partial_y^2 R(y) + (2-3y)\partial_y R(y) + \left[\frac{l^2\omega^2}{1-y} + \left(l^2\omega^2 + \frac{Q}{2}\right)\frac{1}{y} + \frac{m^2 l^2}{4}\right]R(y) = 0. \quad (5)$$

If we define $z = \frac{y-1}{y}$ the equation (5), becomes

$$z(1-z)\partial_z^2 R(z) + \partial_z R(z) + \left[\frac{\omega^2 l^2}{z} - \left(l^2\omega^2 + \frac{Q}{2}\right) - \frac{m^2 l^2}{4(1-z)}\right]R(z) = 0, \quad (6)$$

and the decomposition $R(z) = z^\alpha(1-z)^\beta K(z)$, with

$$\alpha_\pm = \pm i\omega l, \quad (7)$$

$$\beta_\pm = \frac{1}{2}\left(2 \pm \sqrt{4 + m^2 l^2}\right), \quad (8)$$

let us to write (6), as a hypergeometric equation for K

$$z(1-z)K''(z) + [c - (1+a+b)z]K'(z) - abK(z) = 0, \quad (9)$$

where the coefficients are given by

$$a = \frac{1}{2}\left(-1 + 2\alpha + 2\beta \pm \sqrt{1 - 2Q - 4\omega^2 l^2}\right), \quad (10)$$

$$b = \frac{1}{2} \left(-1 + 2\alpha + 2\beta \mp \sqrt{1 - 2Q - 4\omega^2 l^2} \right) , \quad (11)$$

$$c = 1 + 2\alpha . \quad (12)$$

The general solution of the hypergeometric equation (9) is

$$K = C_1 F_1(a, b, c; z) + C_2 z^{1-c} F_1(a - c + 1, b - c + 1, 2 - c; z) , \quad (13)$$

and it has three regular singular points at $z = 0$, $z = 1$. Where $z = \infty$. $F_1(a, b, c; z)$ is a hypergeometric function and C_1 and C_2 are constants. Then, the solution for the radial function $R(z)$ is

$$R(z) = C_1 z^\alpha (1 - z)^\beta F_1(a, b, c; z) + C_2 z^{-\alpha} (1 - z)^\beta F_1(a - c + 1, b - c + 1, 2 - c; z) . \quad (14)$$

So, in the vicinity of the horizon, $z = 0$ and using the property $F(a, b, c, 0) = 1$, the function $R(z)$ behaves as

$$R(z) = C_1 e^{\alpha \ln z} + C_2 e^{-\alpha \ln z} , \quad (15)$$

and the scalar field ψ , for $\alpha = \alpha_-$, can be written in the following way

$$\psi \sim C_1 e^{-i\omega(t+l \ln z)} + C_2 e^{-i\omega(t-l \ln z)} , \quad (16)$$

in which, the first term represents an ingoing wave and the second one an outgoing wave in the black hole. So, by imposing that only ingoing waves exist at the horizon, this fixes $C_2 = 0$. Then, the radial solution becomes

$$R(z) = C_1 e^{\alpha \ln z} (1 - z)^\beta F_1(a, b, c; z) = C_1 e^{-i\omega l \ln z} (1 - z)^\beta F_1(a, b, c; z) . \quad (17)$$

To implement boundary conditions at infinity ($z = 1$), we shall apply the Kummer's formula for the hypergeometric function [30],

$$F_1(a, b, c; z) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1 + (1-z)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_2 . \quad (18)$$

Where,

$$F_1 = F_1(a, b, a+b-c, 1-z) , \quad (19)$$

$$F_2 = F_1(c-a, c-b, c-a-b+1, 1-z) . \quad (20)$$

With this expression the radial function (17) reads

$$R(z) = C_1 e^{-i\omega l \ln z} (1 - z)^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1 + C_1 e^{-i\omega l \ln z} (1 - z)^{c-a-b+\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_2 , \quad (21)$$

and at infinity can be written as

$$R_{asympt.}(z) = C_1 (1 - z)^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + C_1 (1 - z)^{2-\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} . \quad (22)$$

For, $\beta_+ > 2$ the field at infinity vanishes if $(a)|_{\beta_+} = -n$ or $(b)|_{\beta_+} = -n$ for $n = 0, 1, 2, \dots$. For, $\beta_- < 0$ the field at infinity vanishes if $(c-a)|_{\beta_-} = -n$ or $(c-b)|_{\beta_-} = -n$. Therefore, the quasinormal modes are given by

$$\omega_1 = -i \frac{\sqrt{4 + m^2 l^2} + 2n}{4l} - i \frac{2Q + \sqrt{4 + m^2 l^2} + 2n}{4l (\sqrt{4 + m^2 l^2} + 2n + 1)} . \quad (23)$$

Note that four dimensional Topological Lifshitz black hole is stable due to the imaginary part of the QNMs (23), is negative.

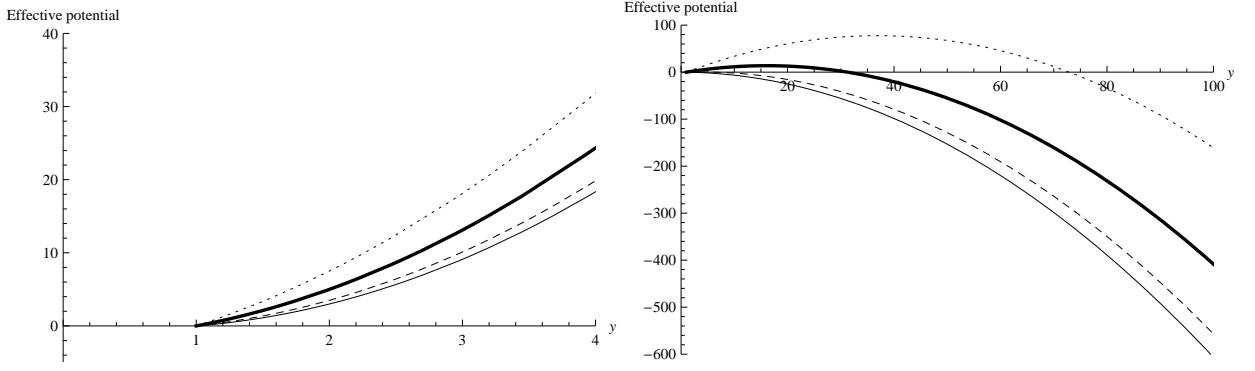


FIG. 1: The effective potential as a function of y for $\xi = 0$ (solid curve), $\xi = 1$ (dashed curve), $\xi = 2$ (thick curve), $\xi = 3$ (dotted curve), $l = 1$, $m = 1.8$ and $m = 1.8i$, respectively.

Now, we consider that the flux vanishes at infinity (or vanishing Neumann boundary condition at infinity) in order to find the QNMs for imaginary masses. Because in asymptotically Lifshitz spacetime, a negative square mass for a scalar field is consistent, in agreement with the analogue to the Breitenlohner-Freedman condition that any effective mass must satisfy in order to have a stable propagation, [31], [32]. Dirichlet boundary condition must lead to the same quasinormal modes for positive square mass from a holographic point of view, but does not lead to any quasinormal modes for negative square mass. However, quasinormal modes can also be obtained by considering Neumann boundary conditions and in the context of conformal field theory, due to the AdS/CFT correspondence [21], the quasinormal modes match with the dual operators, [22].

To illustrate this, we consider a real scalar field in the Euclidean version of metric (1) by performing a Wick rotation $\tau = it$, as in Ref. [23], and considering ingoing waves at the horizon, the bulk scalar field takes the asymptotic form

$$\psi \sim c_1 x^{\Delta_+} \psi_+(\tau, \theta, \phi) + c_2 x^{\Delta_-} \psi_-(\tau, \theta, \phi) , \quad (24)$$

where $x = 1 - z$ and $\psi_{\pm}(\tau, \theta, \phi)$ are sources for a dual field. Δ_{\pm} are roots of $\Delta(\Delta - 4) = m^2 l^2$, given by

$$\Delta_{\pm} = 2 \left(1 \pm \frac{1}{2} \sqrt{4 + m^2 l^2} \right) = 2\beta_{\pm} . \quad (25)$$

The on-shell Euclidean scalar field action is finite for $m^2 l^2 \geq -4$ [23] and [28]. Such as has been shown in Ref. [23], for $-4 < m^2 l^2 < -3$ there are two set of dual operators with dimensions Δ_+ and Δ_- , respectively. Therefore, the second set of quasinormal modes ω_2 matches the dual operators with $\Delta = \Delta_-$ for this range of scalar field mass. Now, we consider the tortoise coordinate

$$\frac{dy^*}{dy} = \frac{l}{y(y-1)} , \quad (26)$$

to analyze the behaviour of the effective potential at infinity. In this way, the radial equation (5), can be written as a Schrödinger equation

$$\left(\frac{d^2}{dy^{*2}} + \omega^2 - V(y) \right) \left(y^{1/2} R(y^*) \right) = 0 , \quad (27)$$

where

$$V(y) = (1-y) \left(\frac{1-y}{4l^2} - \frac{y}{2l^2} - \frac{Q}{2l^2} - \frac{m^2 y}{4} \right) . \quad (28)$$

We observe that the effective potential go to $+\infty$ for $3 + m^2 l^2 > 0$ and go to $-\infty$ for $3 + m^2 l^2 < 0$. See Fig. (1). It is interesting note that in the asymptotic limit $y \rightarrow \infty$ Eq. (27) can be written as a Schrödinger equation with an inverse square potential given by

$$V(y^*) \sim \left(\frac{m^2 l^2}{4} + \frac{3}{4} \right) \frac{1}{y^{*2}} , \quad (29)$$

where the stability of the scalar field is given by $\frac{m^2 l^2}{4} + \frac{3}{4} \geq -\frac{1}{4}$ [33] or $m^2 l^2 \geq -4$. The same analysis has been performed for Lifshitz black hole in New Massive Gravity, [25], where the behavior is similar. Thus, we consider that the flux given by

$$F = \frac{\sqrt{-g} g^{rr}}{2i} (R^* \partial_r R - R \partial_r R^*) , \quad (30)$$

must vanishes at infinity. So, using (21), we obtain that the flux is

$$F = \zeta |C_1|^2 \text{Im} \left(2(\beta - 1) B_1^* B_2 + (\alpha + \gamma) |B_1|^2 (1 - z)^{2\beta - 1} + (\alpha + \eta) |B_2|^2 (1 - z)^{3 - 2\beta} \right) , \quad (31)$$

where

$$\zeta = \frac{l \sinh \theta}{2} , \quad (32)$$

$$\gamma = \frac{ab}{a + b - c} , \quad (33)$$

$$\eta = \frac{(c - a)(c - b)}{c - a - b + 1} , \quad (34)$$

$$B_1 = \frac{\Gamma(c) \Gamma(c - a - b)}{\Gamma(c - a) \Gamma(c - b)} , \quad (35)$$

$$B_2 = \frac{\Gamma(c) \Gamma(a + b - c)}{\Gamma(a) \Gamma(b)} . \quad (36)$$

If $-4 \leq m^2 l^2 \leq 0$, then $1 < \beta_+ < 2$ and $0 < \beta_- < 1$. So, $2\beta - 1 < 0$ for $\beta = \beta_-$ and $3 - 2\beta < 0$ for $\beta = \beta_+$. For, β_- the flux at infinity vanishes if $(c - a)|_{\beta_-} = -n$ or $(c - b)|_{\beta_-} = -n$. For β_+ the flux at infinity vanishes if $(a)|_{\beta_+} = -n$ or $(b)|_{\beta_+} = -n$. These conditions determine the quasinormal modes. In spite of this, these conditions given the same QNMs that we have found (23) previously.

For, $\beta = \beta_+$, the conditions $2\beta_+ - 1 > 0$ and $3 - 2\beta_+ > 0$ are satisfied if $\sqrt{3} < \text{Im}(ml) < 2$. In this case, the flux at infinity vanishes if $B_1 = 0$, i.e., if $(c - a)|_{\beta_+} = -n$ or $(c - b)|_{\beta_+} = -n$. Therefore, the quasinormal modes are

$$\omega_2 = -i \frac{2n - \sqrt{4 + m^2 l^2}}{4l} - i \frac{2Q - \sqrt{4 + m^2 l^2} + 2n}{4l(-\sqrt{4 + m^2 l^2} + 2n + 1)} . \quad (37)$$

The restriction $B_2 = 0$ yields the same QNMs we have previously found (23). In this case, the imaginary part of the QNMs (37) is negative if $\sqrt{3} < \text{Im}(ml) < 2$ for $Q \geq 1/2$ and the imaginary part is negative if $\sqrt{2 + 2Q + 2\sqrt{1 - 2Q}} < \text{Im}(ml) < 2$ for $Q < 1/2$. Therefore, under these conditions the four dimensional Topological Lifshitz black hole is stable.

III. REFLECTION COEFFICIENT, TRANSMISSION COEFFICIENT AND ABSORPTION CROSS SECTION

The reflection and transmission coefficients depend on the behaviour of the radial function both, at the horizon and at the asymptotic infinity and they are defined by

$$R := \left| \frac{F_{\text{asympt}}^{\text{out}}}{F_{\text{asympt}}^{\text{in}}} \right| ; \quad T := \left| \frac{F_{\text{hor}}^{\text{in}}}{F_{\text{asympt}}^{\text{in}}} \right| , \quad (38)$$

where F is the flux (30). The behaviour at the horizon is given by (15), and using (30), we get the flux at the horizon up to an irrelevant factor from the angular part of the solution

$$F_{\text{hor}}^{\text{in}} = -\frac{1}{2} \omega l^2 |C_1|^2 . \quad (39)$$

To obtain the asymptotic behaviour of $R(z)$, we use $x = 1 - z$ in (6) and we consider the asymptotic infinity, i.e. $x \rightarrow 0$. Thus, we obtain the following equation

$$x^2 R''(x) - x R'(x) + \left(-\frac{m^2 l^2}{4} + x \left(\frac{m^2 l^2}{4} - \frac{Q}{2} \right) \right) R(x) = 0 . \quad (40)$$

Whose solution is

$$R_{asymp.}(x) = E_1 B \Gamma(1 - A) J_{-A} \left(2\sqrt{Bx} \right) x + E_2 B \Gamma(1 + A) J_A \left(2\sqrt{Bx} \right) x , \quad (41)$$

where

$$A = 2\sqrt{1 + \frac{m^2 l^2}{4}} , \quad B = \frac{m^2 l^2}{4} - \frac{Q}{2} , \quad (42)$$

and using the expansion of the Bessel function [30]

$$J_n(q) = \frac{q^n}{2^n \Gamma(n+1)} \left\{ 1 - \frac{q^2}{2(2n+2)} + \dots \right\} , \quad (43)$$

for $q \ll 1$, we find the asymptotic infinity solution in the polynomial form

$$R_{asymp.}(x) = E_1 B^{1-A/2} x^{1-A/2} + E_2 B^{1+A/2} x^{1+A/2} , \quad (44)$$

for $2\sqrt{Bx} \ll 1$. Note that $1 - A/2 = \beta_-$ and (44) match with (22) with

$$E_1 B^{1-A/2} = C_1 \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} = C_1 B_1 , \quad (45)$$

$$E_2 B^{1+A/2} = C_1 \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} = C_1 B_2 . \quad (46)$$

The radial function (44) is regular at infinity if $1 - \frac{A}{2} \geq 0$ or $-4 \leq m^2 l^2 \leq 0$, and A cannot be an integer, due to $a + b - c = -A$, for $\beta = \beta_-$, and $c - a - b = -A$, for $\beta = \beta_+$ and the gamma function is singular at that point. The first condition is in agreement with the analogue to Breitenlohner-Freedman condition. This is, that any effective mass must be satisfied in order to have a stable propagation, [31] and [32].

Therefore, the flux (30) at the asymptotic region (44) is given by

$$F_{asymp} \approx -(1 - \beta) l |C_1|^2 \text{Im}[B_2 B_1^*] . \quad (47)$$

up to an irrelevant factor from the angular part of the solution. In order to characterize the fluxes, we split up the coefficients B_1 and B_2 in terms of the incoming and outgoing coefficients, B_{in} and B_{out} , respectively. We define $B_1 = B_{in} + B_{out}$ and $B_2 = ih(B_{out} - B_{in})$ with h being a dimensionless constant which it will be assumed to be independent of the energy ω [34], [35], [36], [37], [38], [39] and [26]. In this way, we can write the asymptotic flux (47) as

$$F_{asymp} \approx (1 - \beta) l h |C_1|^2 [|B_{in}|^2 - |B_{out}|^2] . \quad (48)$$

Therefore, the reflection and transmission coefficients are given by

$$R = \frac{|B_{out}|^2}{|B_{in}|^2} , \quad (49)$$

$$T = \frac{\omega l}{2|1 - \beta||h||B_{in}|^2} , \quad (50)$$

and the absorption cross section σ_{abs} , becomes

$$\sigma_{abs} = \frac{l}{2|1 - \beta||h||B_{in}|^2} , \quad (51)$$

where, the coefficients $|B_{\text{in}}|^2$ and $|B_{\text{out}}|^2$ are given by

$$|B_{\text{in}}|^2 = \frac{1}{4} \left\{ |B_1|^2 + \frac{1}{h^2} |B_2|^2 - \frac{2}{h} \text{Im}[B_2 B_1^*] \right\}, \quad (52)$$

$$|B_{\text{out}}|^2 = \frac{1}{4} \left\{ |B_1|^2 + \frac{1}{h^2} |B_2|^2 + \frac{2}{h} \text{Im}[B_2 B_1^*] \right\}. \quad (53)$$

The poles of the transmission coefficient are given by $|B_{\text{in}}|^2 = 0$. Using equation (52) and solving for h , we find the following relations must hold for the existence of a pole

$$h = \frac{\text{Im}(B_1^* B_2)}{|B_1|^2}, \quad \text{Im}(B_1^* B_2) = \pm |B_1| |B_2|. \quad (54)$$

Solving these equations for ω it could be possible to obtain a set of frequencies, however it is a difficult task. An interesting question is to see if these poles coincide with the quasinormal modes ω_1 obtained in section II. Due that ω_1 are purely imaginary, the constants $\alpha, \beta, a, b, c, B_1$ and B_2 are real when evaluated in these modes, therefore $\text{Im}(B_1^* B_2) = 0$, and from equation (52) we see that for these QNMs to coincide with the poles of the transmission coefficient, the following relation must hold

$$|B_1|^2 + \frac{1}{h^2} |B_2|^2 = 0, \quad (55)$$

but this relation implies $|B_1| = |B_2| = 0$. However $|B_1|$ and $|B_2|$ can't be both zero. Therefore the QNMs ω_1 doesn't correspond to the poles of the transmission coefficient.

It is known that the constant h can be chosen so that the absorption cross section can be expressed by the area of horizon in the zero-frequency limit [34] and [40]. Also, it can be chosen to obtain the usual value of the Hawking temperature [35] or in such a way so that the sum of the reflection coefficient and the transmission coefficient be unity [41]. For negative h we obtain that $R + T = 1$ and for positive h this relation doesn't always hold, see Fig. (2) and (3). Based on this, in this work we take arbitrarily, but without loss of generality $h = -1$ in all the pictures in such

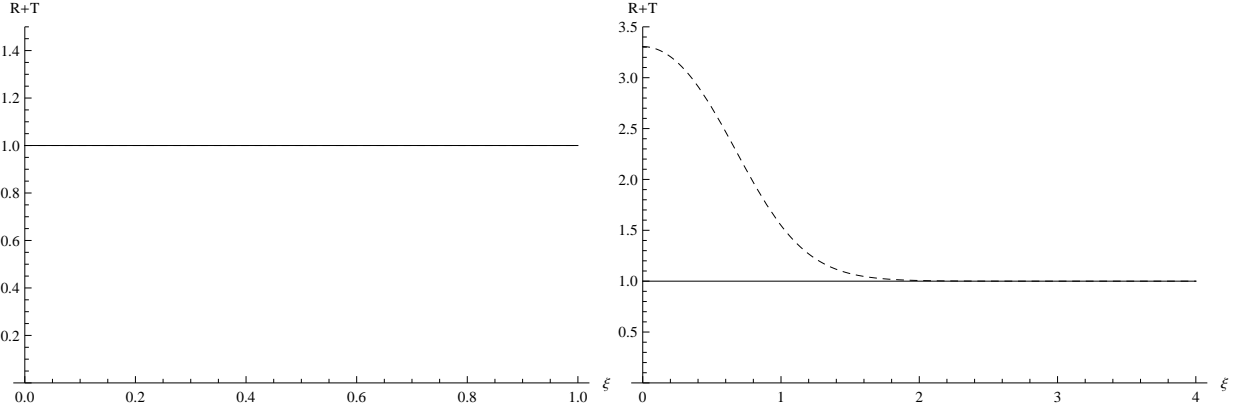


FIG. 2: $R+T$ as a function of ξ ; for $h = -1$ and $h = 1$, respectively. Solid curve for $\omega = 0$ and dashed curve for $\omega = 1$; $m = i$ and $l = 1$.

way to ensure that $R + T = 1$ for any angular momentum, the qualitative behavior of the coefficients is the same for any negative h .

Now, we will carry out a numerical analysis of the reflection (49), and transmission coefficients (50), for a four dimensional topological Lifshitz black hole. We consider, without loss of generality, $l = 1$ and we fix $h = -1$. Note that in (52) if h is negative, the coefficients are regular and then we plot them for $\xi = 0, 0.5, 1, 1.5$. See Figs. (4, 5), for the reflection, transmission coefficients and the greybody factors, for positive scalar field mass ($m = 1$) and imaginary scalar field mass ($m = i$), respectively. Here, we observe that in the zero-frequency limit, there is a range of values of ξ which contribute to the absorption cross section, see Fig. (6), as it happens in [38], [39], [42] and [26]. Also, we observed in the low frequency limit, that the reflection and transmission coefficients show a minimum and a maximum. Therefore, the coefficients have two branches in the reflection case, decreasing for low frequencies and then

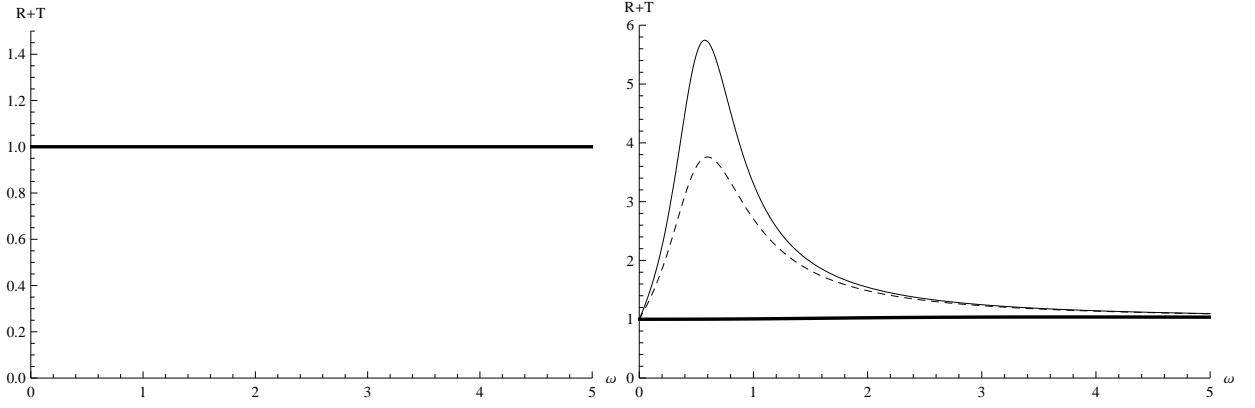


FIG. 3: $R+T$ as a function of ω , for $h = -1$ and $h = 1$, respectively. Solid curve for $\xi = 0$, dashed curve for $\xi = 0.5$ and thick curve for $\xi = 2$; $m = i$ and $l = 1$.

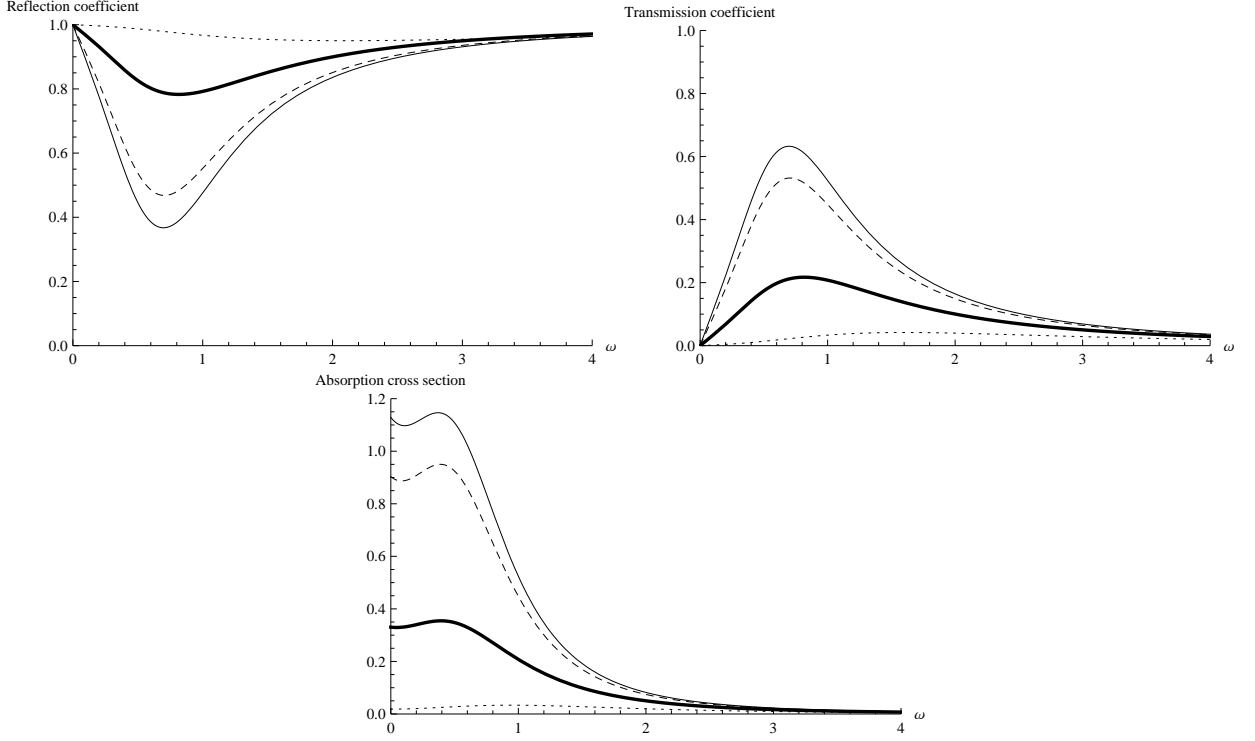


FIG. 4: The reflection coefficient, the transmission coefficient and the absorption cross section as a function of ω ; for $\xi = 0$ (solid curve), $\xi = 0.5$ (dashed curve), $\xi = 1$ (thick curve), $\xi = 1.5$ (dotted curve), $m = 1$, $l = 1$ and $h = -1$.

increasing. In the transmission case the behaviour is opposite, they are increasing and then decreasing, in such a way that $R + T = 1$ occurs in all cases. In order to study the effect of the mass of the scalar field, we plot the reflection and transmission coefficient and the absorption cross section for the mode with lowest angular momentum $\xi = 0$, with $l = 1$ and $h = -1$ in Figs. (7, 8), and we consider positive scalar field mass ($m = 1, 2, 3, 4$) and imaginary scalar field mass ($m = i, 1.2i, 1.4i, 1.8i$), respectively. It is worth to mention that the coefficients are regular for positive scalar field mass, but (44) is not regular for positive scalar field mass. Essentially, we found the same general behaviour for the different values of m and the only difference is a shift in the location of the minimum or maximum of the reflection and transmission coefficients, respectively. We observe that the absorption cross is decreasing if the parameter m is increasing in the low frequency limit for real scalar field mass, Fig (7). We also can observe the existence of one optimal frequency to transfer energy out of the bulk and the absorption cross section vanishes in the low frequency

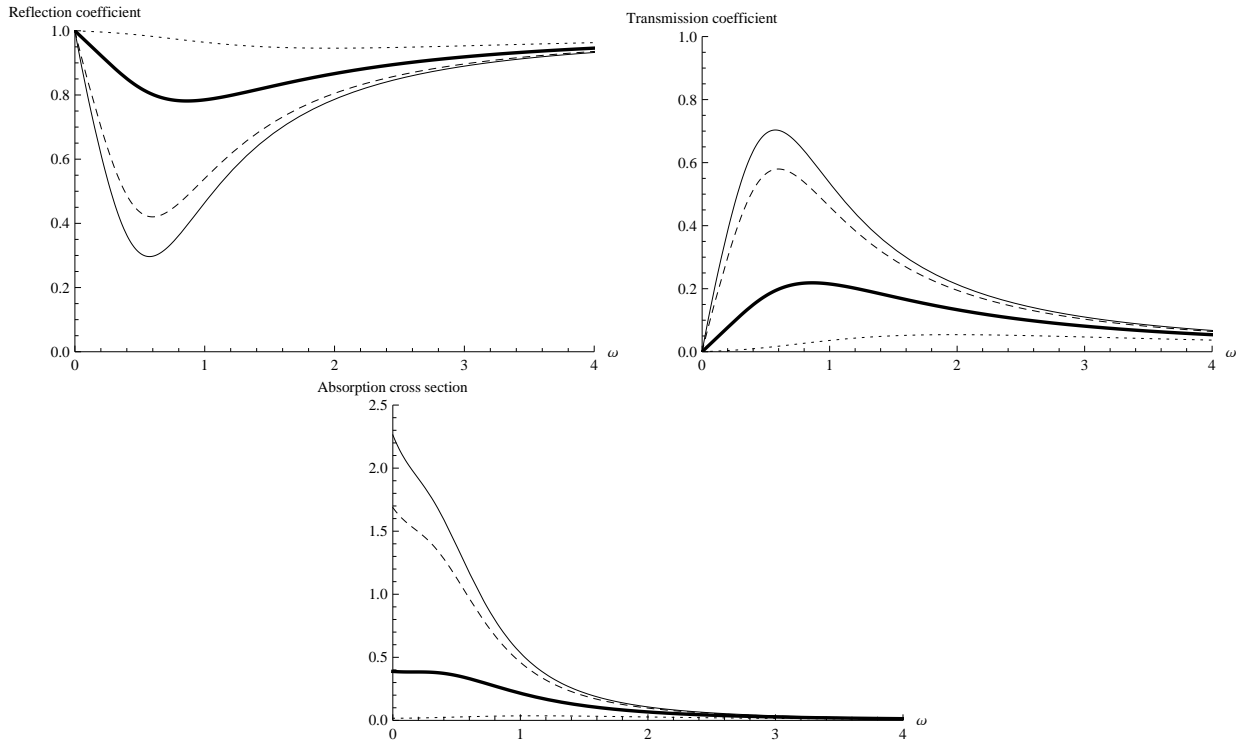


FIG. 5: The reflection coefficient, the transmission coefficient and the absorption cross section as a function of ω ; for $\xi = 0$ (solid curve), $\xi = 0.5$ (dashed curve), $\xi = 1$ (thick curve), $\xi = 1.5$ (dotted curve), $m = i$, $l = 1$ and $h = -1$.

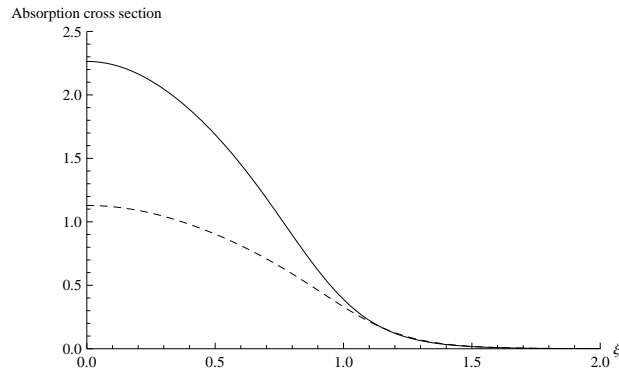


FIG. 6: Absorption cross section as a function of ξ ; for $m = i$ (solid curve), $m = 1$ (dashed curve), $l = 1$, $\omega = 0$ and $h = -1$.

limit, for $\xi = 0$ and $m \rightarrow 0$.

IV. CONCLUSIONS

The Lifshitz black hole is a very interesting static solution of gravity theories which asymptotically approach to Lifshitz spacetimes. In this work we have studied four dimensional Topological Lifshitz black hole under scalar perturbations. We have considered Dirichlet and Neumann boundary conditions and we have found that the quasinormal modes are purely imaginary i.e., there is only damped modes, which guaranteed the stability of Lifshitz black hole due to the imaginary part of the QNMs is negative. Also, we have computed analytically the reflection and transmission coefficients and the absorption cross section and we have found that there is a range of modes with high angular momentum which contributes to the absorption cross section in the low frequency limit. Furthermore, in this limit,

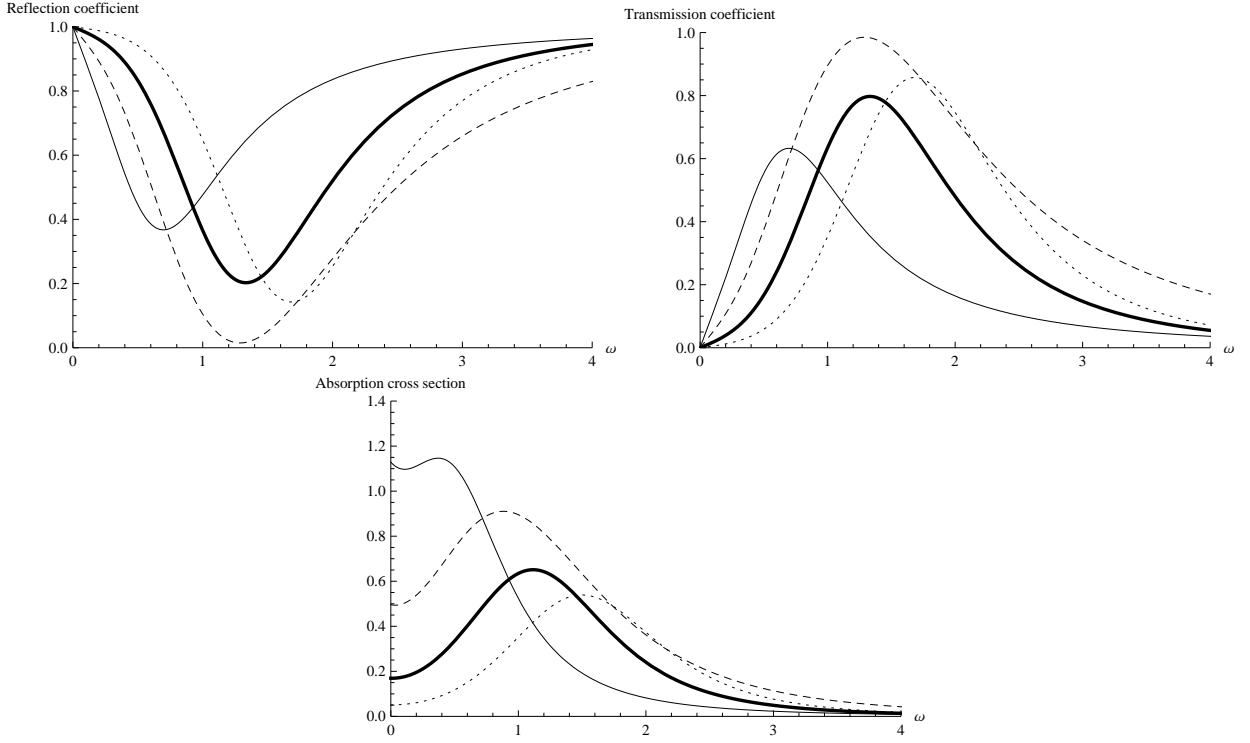


FIG. 7: The reflection coefficient, the transmission coefficient and the absorption cross section as a function of ω ; for $m = 1$ (solid curve), $m = 2$ (dashed curve), $m = 3$ (thick curve), $m = 4$ (dotted curve), $\xi = 0$, $l = 1$ and $h = -1$.

the absorption cross section decreases if the scalar field mass increase, for a real scalar field mass.

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- [1] S. A. Hartnoll, “Lectures on holographic methods for condensed matter physics,” *Class. Quant. Grav.* **26**, 224002 (2009) [arXiv:0903.3246 [hep-th]].
 - [2] R. B. Mann, “Lifshitz Topological Black Holes,” *JHEP* **0906**, 075 (2009) [arXiv:0905.1136 [hep-th]].
 - [3] T. Regge and J. A. Wheeler, “Stability Of A Schwarzschild Singularity,” *Phys. Rev.* **108**, 1063 (1957).
 - [4] F. J. Zerilli, “Gravitational field of a particle falling in a schwarzschild geometry analyzed in tensor harmonics,” *Phys. Rev. D* **2**, 2141 (1970).
 - [5] F. J. Zerilli, “Effective Potential For Even Parity Regge-Wheeler Gravitational Perturbation Equations,” *Phys. Rev. Lett.* **24**, 737 (1970).
 - [6] K. D. Kokkotas and B. G. Schmidt, “Quasi-normal modes of stars and black holes,” *Living Rev. Rel.* **2**, 2 (1999) [arXiv:gr-qc/9909058].
 - [7] H. P. Nollert, “TOPICAL REVIEW: Quasinormal modes: the characteristic ‘sound’ of black holes and neutron stars,” *Class. Quant. Grav.* **16**, R159 (1999).
 - [8] E. Berti, V. Cardoso and A. O. Starinets, “Quasinormal modes of black holes and black branes,” *Class. Quant. Grav.* **26**, 163001 (2009) [arXiv:0905.2975 [gr-qc]].
 - [9] R. A. Konoplya and A. Zhidenko, “Quasinormal modes of black holes: from astrophysics to string theory,” *Rev. Mod. Phys.* **83**, 793 (2011) [arXiv:1102.4014 [gr-qc]].
 - [10] A.A. Starobinsky and S.M. Churilov, *Sov. Phys.- JETP* **38**, 1 (1974)

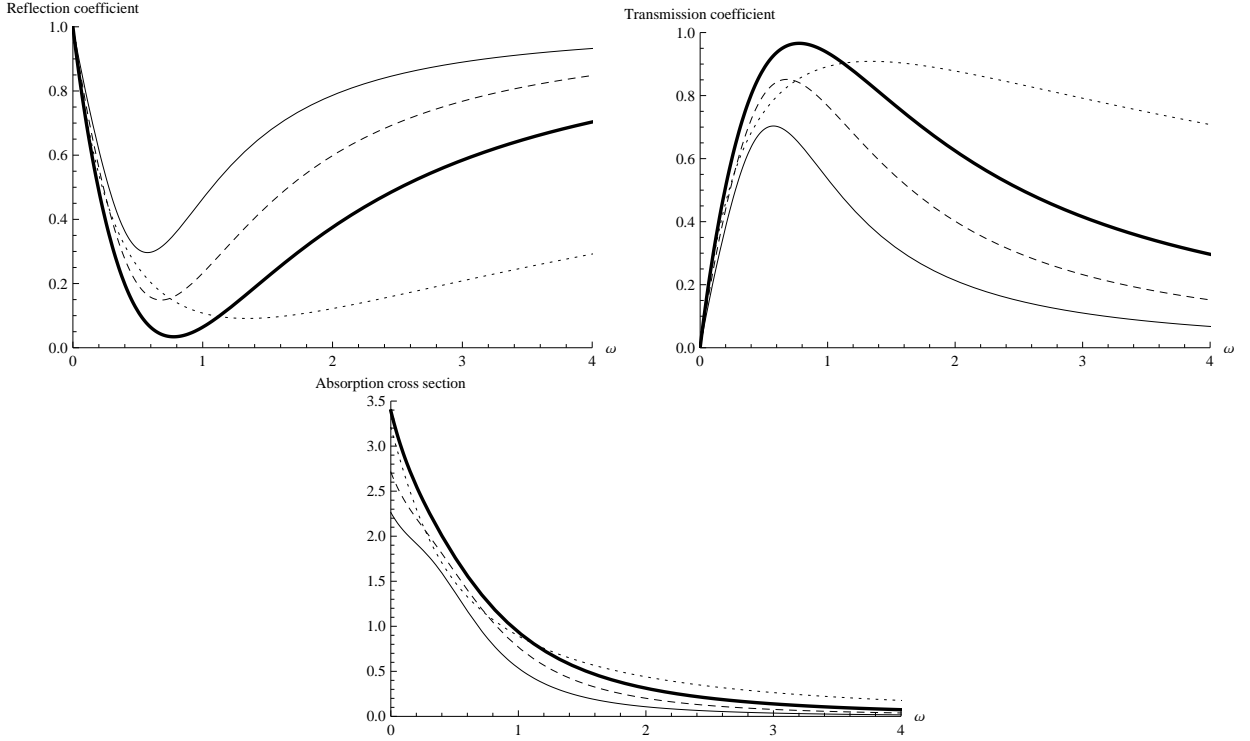


FIG. 8: The reflection coefficient, the transmission coefficient and the absorption cross section as a function of ω ; for $m = i$ (solid curve), $m = 1.2i$ (dashed curve), $m = 1.4i$ (thick curve), $m = 1.8i$ (dotted curve), $\xi = 0$, $l = 1$ and $h = -1$.

- [11] G. W. Gibbons, “Vacuum Polarization and the Spontaneous Loss of Charge by Black Holes,” *Commun. Math. Phys.* **44** (1975) 245.
- [12] D. N. Page, “Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole,” *Phys. Rev. D* **13** (1976) 198.
- [13] W. G. Unruh, “Absorption Cross-Section of Small Black Holes,” *Phys. Rev. D* **14** (1976) 3251.
- [14] A. Dhar, G. Mandal and S. R. Wadia, “Absorption versus decay of black holes in string theory and T symmetry,” *Phys. Lett. B* **388** (1996) 51 [hep-th/9605234].
- [15] B. Kol and A. Rajaraman, “Fixed scalars and suppression of Hawking evaporation,” *Phys. Rev. D* **56** (1997) 983 [hep-th/9608126].
- [16] G. W. Gibbons and A. R. Steif, “Anomalous fermion production in gravitational collapse,” *Phys. Lett. B* **314** (1993) 13 [gr-qc/9305018].
- [17] S. Hod, “Bohr’s correspondence principle and the area spectrum of quantum black holes,” *Phys. Rev. Lett.* **81**, 4293 (1998) [arXiv:gr-qc/9812002].
- [18] G. Kunstatter, “d-dimensional black hole entropy spectrum from quasi-normal modes,” *Phys. Rev. Lett.* **90**, 161301 (2003) [arXiv:gr-qc/0212014].
- [19] M. Maggiore, “The physical interpretation of the spectrum of black hole quasinormal modes,” *Phys. Rev. Lett.* **100**, 141301 (2008) [arXiv:0711.3145 [gr-qc]].
- [20] G. T. Horowitz and V. E. Hubeny, “Quasinormal modes of AdS black holes and the approach to thermal equilibrium,” *Phys. Rev. D* **62**, 024027 (2000) [arXiv:hep-th/9909056].
- [21] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200].
- [22] D. Birmingham, I. Sachs and S. N. Solodukhin, “Conformal field theory interpretation of black hole quasinormal modes,” *Phys. Rev. Lett.* **88**, 151301 (2002) [hep-th/0112055].
- [23] S. Kachru, X. Liu and M. Mulligan, “Gravity Duals of Lifshitz-like Fixed Points,” *Phys. Rev. D* **78**, 106005 (2008) [arXiv:0808.1725 [hep-th]].
- [24] B. Cuadros-Melgar, J. de Oliveira and C. E. Pellicer, “Stability Analysis and Area Spectrum of 3-Dimensional Lifshitz Black Holes,” *Phys. Rev. D* **85**, 024014 (2012) [arXiv:1110.4856 [hep-th]].
- [25] T. Moon and Y. S. Myung, “Absorption cross section in Lifshitz black hole,” arXiv:1205.2317 [hep-th].
- [26] S. Lepe, J. Lorca, F. Pena and Y. Vasquez, “Scalar field scattering by a Lifshitz black hole under a non-minimal coupling,” arXiv:1205.4460 [hep-th].
- [27] P. A. Gonzalez, J. Saavedra and Y. Vasquez, “Quasinormal modes and Stability Analysis for 4-dimensional Lifshitz Black

- Hole,” arXiv:1201.4521 [gr-qc].
- [28] A. Giacomini, G. Giribet, M. Leston, J. Oliva and S. Ray, “Scalar field perturbations in asymptotically Lifshitz black holes,” arXiv:1203.0582 [hep-th].
 - [29] Y. S. Myung and T. Moon, “Quasinormal frequencies and thermodynamic quantities for the Lifshitz black holes,” arXiv:1204.2116 [hep-th].
 - [30] M. Abramowitz and A. Stegun, Handbook of Mathematical functions, (Dover publications, New York, 1970).
 - [31] P. Breitenlohner and D. Z. Freedman, “Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity,” Phys. Lett. B **115**, 197 (1982);
 - [32] P. Breitenlohner and D. Z. Freedman, “Stability In Gauged Extended Supergravity,” Annals Phys. **144**, 249 (1982).
 - [33] K. M. Case, “Singular potentials,” Phys. Rev. **80**, 797 (1950).
 - [34] D. Birmingham, I. Sachs and S. Sen, “Three-dimensional black holes and string theory,” Phys. Lett. B **413**, 281 (1997) [arXiv:hep-th/9707188].
 - [35] W. T. Kim and J. J. Oh, “Dilaton driven Hawking radiation in AdS(2) black hole,” Phys. Lett. B **461**, 189 (1999) [arXiv:hep-th/9905007].
 - [36] J. J. Oh and W. Kim, “Absorption Cross Section in Warped AdS₃ Black Hole,” JHEP **0901**, 067 (2009) [arXiv:0811.2632 [hep-th]].
 - [37] H. C. Kao and W. Y. Wen, “Absorption cross section in warped AdS₃ black hole revisited,” JHEP **0909**, 102 (2009) [arXiv:0907.5555 [hep-th]].
 - [38] P. Gonzalez, C. Campuzano, E. Rojas and J. Saavedra, “Greybody factors for topological massless black holes,” JHEP **1006**, 103 (2010) [arXiv:1003.2753 [gr-qc]].
 - [39] P. Gonzalez, E. Papantonopoulos and J. Saavedra, “Chern-Simons black holes: scalar perturbations, mass and area spectrum and greybody factors,” JHEP **1008** (2010) 050 [arXiv:1003.1381 [hep-th]].
 - [40] S. R. Das, G. W. Gibbons and S. D. Mathur, “Universality of low energy absorption cross sections for black holes,” Phys. Rev. Lett. **78**, 417 (1997) [arXiv:hep-th/9609052].
 - [41] S. W. Kim, W. T. Kim and J. J. Oh, “Decay Rate and Low Energy Near Horizon Dynamics of Acoustic Black Holes,” Phys. Lett. B **608**, 10 (2005) [arXiv:gr-qc/0409003].
 - [42] P. A. Gonzalez and J. Saavedra, “Comments on absorption cross section for Chern-Simons black holes in five dimensions,” Int. J. Mod. Phys. A **26**, 3997 (2011) [arXiv:1104.4795 [gr-qc]].